Machine Learning for Flux Regression in Discrete Fracture Networks

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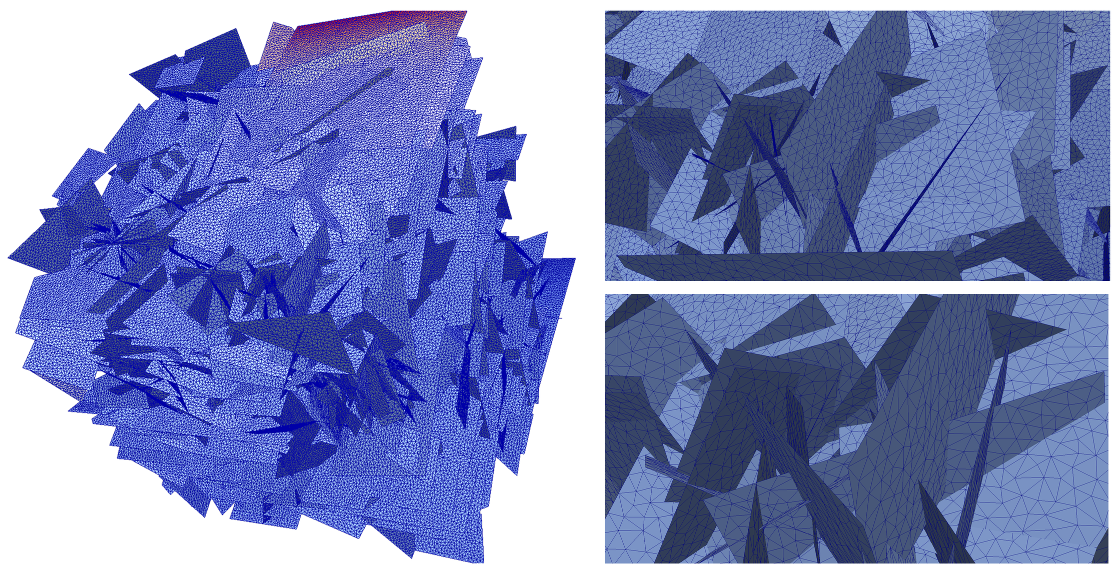
Uncertainty in Discrete Fracture Networks (DFN)

Characterization of **flow and transport through fractured media** in the subsurface is a crucial issue in many applications (e.g. in oil and gas extraction or avoidance of drinking water pollution due to industrial waste). Then models such as **Discrete Fracture Networks (DFN)**, that can accurately simulate flow through networks of subsurface fractures, are required.



Major issues:

- **Building mesh on intersecting fractures** (non- conforming meshes);
- Lack of full deterministic data → **DFN stochastically generated** using probabilistic distribution for generating:
  - **Geometrical properties** (e.g. fracture density, size, position, orientation, ...);
  - **Hydrogeological properties** such as **transmissivity** (parameter that characterizes flow facilitation through a given fracture). We suppose distribution of fracture transmissivities κ_i is:

$$\log_{10} \kappa_i \sim \mathcal{N}(-5, 1)$$

Numerical solution:

- Non-conforming meshes generated independently for each fracture;
- **Strongly parallel approach** (fracture-local problems solved within a gradient-based approach);
- Different choices available for space discretization (FEM, XFEM, VEM).

Uncertainty and quantities of interest (QoI):

Given a probability space $S = (\Omega, \mathcal{A}, \mathbb{P})$ and an outcome $\omega \in \Omega$, let $w(\omega)$ be solution of our mathematical problem and $\phi(\omega) = g(w(\omega))$ be the application's **QoI** (e.g. the overall flux flowing through the network).

Main target: evaluates moments of ϕ via non-intrusive methods:

$$\mathbb{E}[\phi] = \int_\Omega \phi(\omega)\, d\mathbb{P}\,, \quad \sigma^2[\phi] = \int_\Omega (\phi(\omega) - \mathbb{E}[\phi])^2\, d\mathbb{P}\,.$$

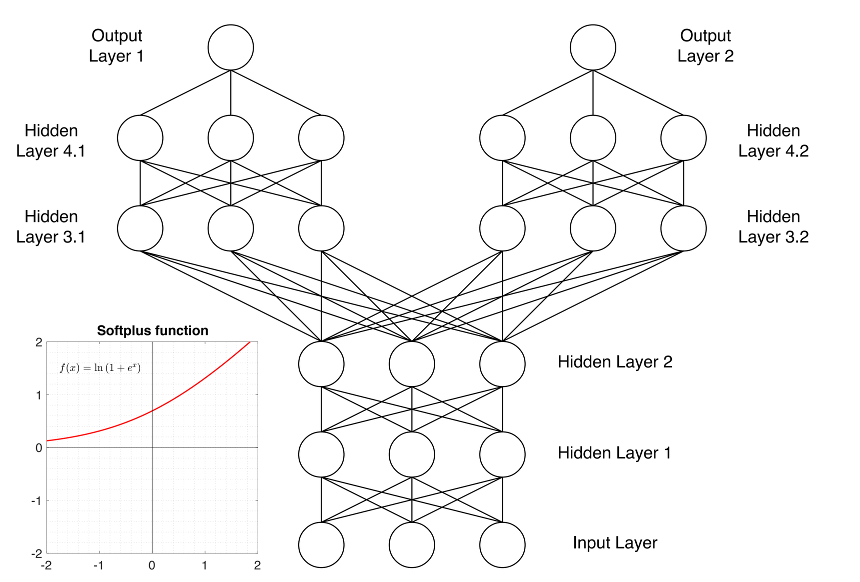
Vector Valued Regression in DFNs

Since QoI's moments evaluation needs lot of simulations, usage of **Neural Networks (NN)** is considered, to speed up this process (through **QoI regression**) and then as an **alternative model reduction method**.

DFNs with **fixed and deterministic geometry** are taken into account to deal with smooth QoI. Let $\hat{N}$ be the total number of fractures in the DFN and let $\hat{m}$ be the total number of fractures with flow exiting the network. Let κ_i be the transmissivity of i-th fracture.

In regression problems considered, we want to **predict** $m \leq \hat{m}$ **exiting flows** of the given DFN, **varying** (without loss of generality) **transmissivities** of first $n \leq \hat{N}$ fractures (and supposing others fixed to value $\kappa = 10^{-5}$). More precisely we want to build a **vector valued regression** model that, for each transmissivities vector $\boldsymbol{k} = [\kappa_1, \dots, \kappa_n]^\top$ with corresponding fluxes $\boldsymbol{\phi} = [\varphi_1, \dots, \varphi_m]^\top$, returns a vector of estimated fluxes:

$$\hat{\boldsymbol{\phi}} = [\hat{\varphi}_1, \dots, \hat{\varphi}_m]^\top \quad s.t. \quad \hat{\boldsymbol{\phi}} \approx \boldsymbol{\phi}$$



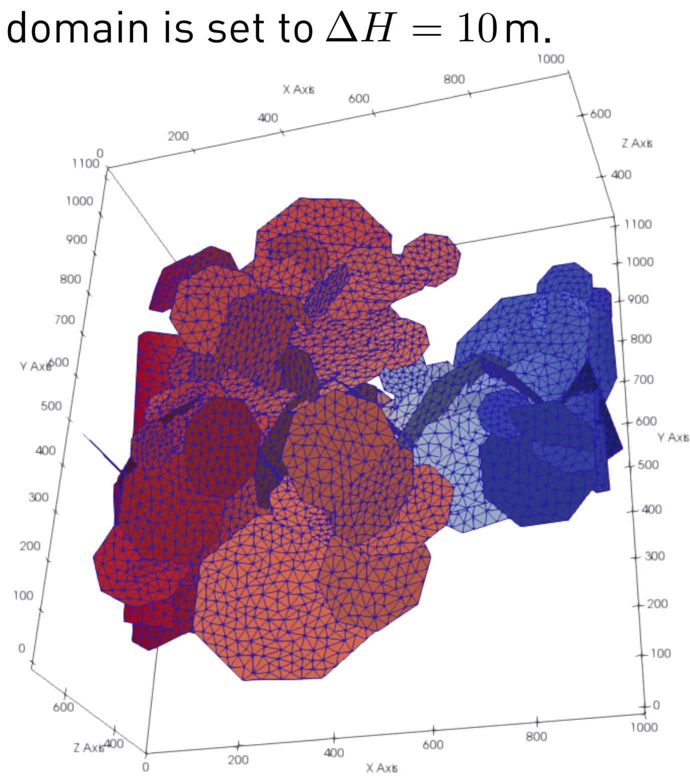
Example of NN for vector valued regression for flux prediction $(n = 3, \quad m = 2, \quad d = 2)$.

NN architecture:

- Input layer of n softplus units;
- d shared layers of n softplus units;
- d task-specific layers of n softplus units for each one of the m outputs;
- Output layer of m units (linked to its last task-specific layer).

Numerical Experiments - Models and Performance Comparisons

We study **NN regression models on a given DFN** with fixed and deterministic geometry consisting of **107 fractures**, in a cubic domain (1000 meters wide) and pressure difference between entering and exiting flow surfaces of domain is set to $\Delta H = 10$m.



Error measures:

- Absolute Error:

$$\boldsymbol{e}_a = [|\hat{\varphi}_1 - \varphi_1|, \dots, |\hat{\varphi}_m - \varphi_m|]^\top$$

- Relative Error:

$$\boldsymbol{e}_r = \left[\frac{|\hat{\varphi}_1 - \varphi_1|}{|\varphi_1|}, \dots, \frac{|\hat{\varphi}_m - \varphi_m|}{|\varphi_m|}\right]^\top$$

- Weighted Absolute Error:

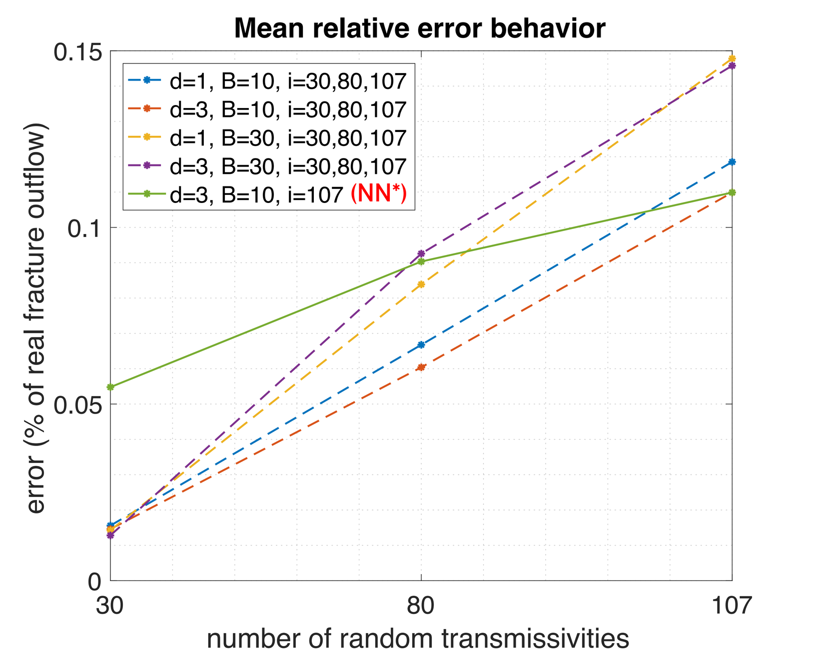
$$\boldsymbol{e}_w = \frac{\boldsymbol{e}_a}{\sum_{i=1}^{\hat{m}} \varphi_i}$$

NN parameters:

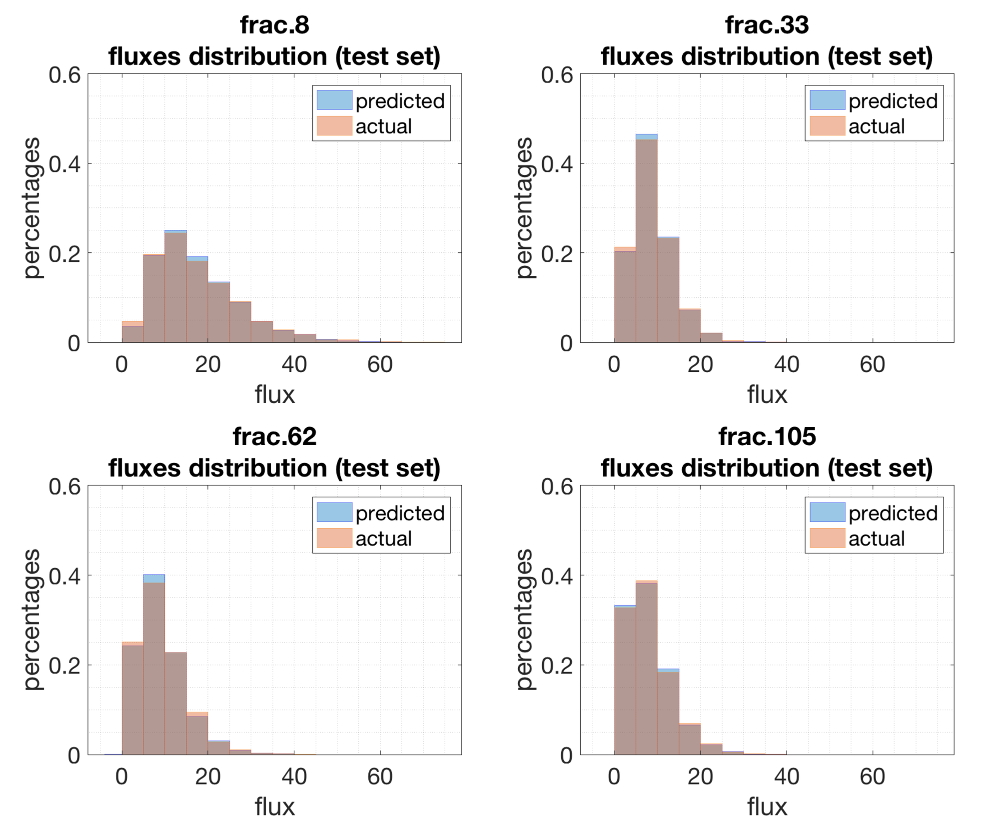
- $m = 4$ fixed (fractures **{8, 33, 62, 105}**; $\hat{m} = 7$);
- $n = 30, 80, 107$;
- $d = 1, 3$;

Training parameters:

- E = 1000 (maximum training epochs);
- B = 10, 30 (mini-batch size for training);
- P = 150 (Early stopping patience parameter).

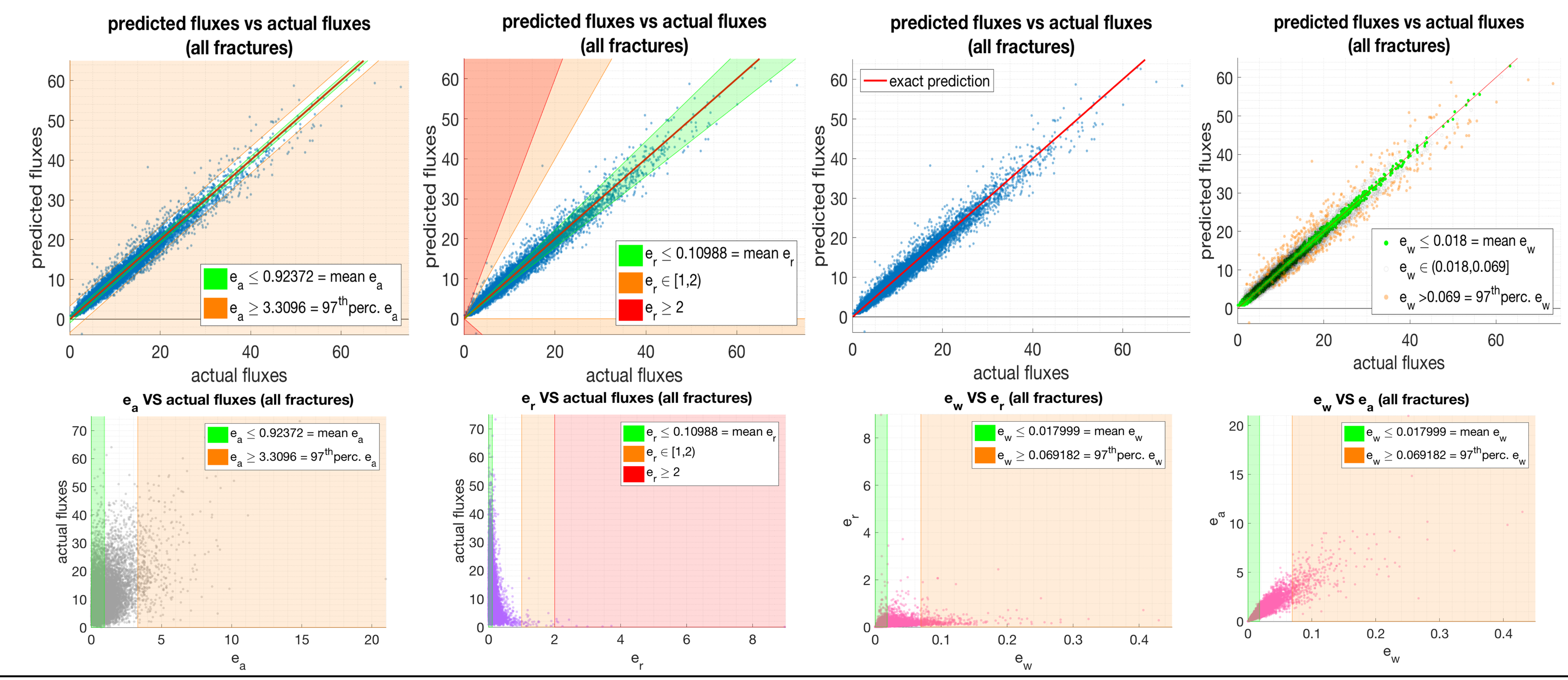


NN* Analysis – Predictions



Fracture	$\mathbb{E}[\varphi]\,(mm^2s^{-1})$	$\mathbb{E}[\hat{\varphi}]\,(mm^2s^{-1})$
8	~ 17.445	~ 17.486
33	~ 8.838	~ 8.796
62	~ 9.165	~ 9.055
105	~ 8.006	~ 7.932
Fracture	$\sigma[\varphi]\,(mm^2s^{-1})$	$\sigma[\hat{\varphi}]\,(mm^2s^{-1})$
8	~ 9.843	~ 9.521
33	~ 4.719	~ 4.533
62	~ 5.579	~ 5.399
105	~ 5.213	~ 5.093

	$e_a\,(mm^2s^{-1})$	e_r	e_w
Mean Val.	~ 0.924	~ 0.110	~ 0.018
97^{th} perc.	~ 3.310	~ 0.385	~ 0.069
max	~ 20.994	~ 8.980	~ 0.430



Conclusions and future work

We built **vector valued NNs for flux regression** problems in DFNs and studied their performances varying parameters, with particular attention to NN* network. **NN* has good performances** (very few serious prediction mistakes) especially considering distribution and moments of QoI considered.

Future works will improve NN architectures and tune parameters for training. A new dataset creation criterion, using homogeneous quality simulation (**adaptive methods**), will be considered.

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